Q1- Two coins lie 1.5 m apart on a table. They carry identical charges. Approximately how large is the charge on each if a coin experiences a force of 2 N?

The diameter of a coin is small compared to the 1.5 m separation. We may therefore approximate the coins as point charges.

Coulomb's Law : $F_E = K q_1 q_2 / r^2$

$$q_1 q_2 = q^2 = \frac{F_E r^2}{k} = \frac{(2\ \text{N})(1.5\ \text{m})^2}{9 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2} = 5 \times 10^{-10}\ \text{C}^2$$

from which $q = 2 \times 10^{-5}$ C.

Q2- A helium nucleus has charge $+2e$, and a neon nucleus $+10e$, where e is the quantum of charge, 1.60×10^{-19} C. Find the repulsive force exerted on one by the other when they are 3.0 nanometers ($1\ \text{nm} = 10^{-9}$ m) apart. Assume the system to be in vacuum.

Nuclei have radii of order 10^{-15} m. We can assume them to be point charges in this case. Then

$$F_E = k\frac{qq'}{r^2} = (9.0 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{(2)(10)(1.6 \times 10^{-19}\ \text{C})^2}{(3.0 \times 10^{-9}\ \text{m})^2} = 5.1 \times 10^{-10}\ \text{N} = 0.51\ \text{nN}$$

Q3- In the Bohr model of the hydrogen atom, an electron ($q = -e$) circles a proton ($q' = e$) in an orbit of radius 5.3×10^{-11} m. The attraction of the proton for the electron furnishes the centripetal force needed to hold the electron in orbit. Find (*a*) the force of electrical attraction between the particles and (*b*) the electron's speed. The electron mass is 9.1×10^{-31} kg.

(*a*)
$$F_E = k\frac{qq'}{r^2} = (9.0 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{(1.6 \times 10^{-19}\ \text{C})^2}{(5.3 \times 10^{-11}\ \text{m})^2} = 8.2 \times 10^{-8}\ \text{N} = 82\ \text{nN}$$

(*b*) The force found in (*a*) is the centripetal force, mv^2/r. Therefore,

$$8.2 \times 10^{-8}\ \text{N} = \frac{mv^2}{r}$$

from which

$$v = \sqrt{\frac{(8.2 \times 10^{-8}\ \text{N})(r)}{m}} = \sqrt{\frac{(8.2 \times 10^{-8}\ \text{N})(5.3 \times 10^{-11}\ \text{m})}{9.1 \times 10^{-31}\ \text{kg}}} = 2.2 \times 10^6\ \text{m/s}$$

Q4- Find the ratio of the Coulomb electric force F_E to the gravitational force F_G between two electrons in vacuum.

From Coulomb's Law and Newton's Law of gravitation,

$$F_E = k\frac{q^2}{r^2} \qquad \text{and} \qquad F_G = G\frac{m^2}{r^2}$$

Therefore
$$\frac{F_E}{F_G} = \frac{kq^2/r^2}{Gm^2/r^2} = \frac{kq^2}{Gm^2}$$
$$= \frac{(9.0 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(1.6 \times 10^{-19}\ \text{C})^2}{(6.67 \times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(9.1 \times 10^{-31}\ \text{kg})^2} = 4.2 \times 10^{42}$$

As you can see, the electric force is much stronger than the gravitational force.

Q5- Compute (*a*) the electric field E in air at a distance of 30 cm from a point charge $q_1 = 5.0 \times 10^{-9}$ C, (*b*) the force on a charge $q_2 = 4.0 \times 10^{-10}$ C placed 30 cm from q_1, and (*c*) the force on a charge $q_3 = -4.0 \times 10^{-10}$ C placed 30 cm from q_1 (in the absence of q_2).

(*a*)
$$E = k\frac{q_1}{r^2} = (9.0 \times 10^9\ \mathrm{N \cdot m^2/C^2})\frac{5.0 \times 10^{-9}\ \mathrm{C}}{(0.30\ \mathrm{m})^2} = 0.50\ \mathrm{kN/C}$$

directed away from q_1.

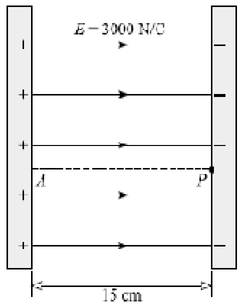
(*b*)
$$F_E = Eq_2 = (500\ \mathrm{N/C})(4.0 \times 10^{-10}\ \mathrm{C}) = 2.0 \times 10^{-7}\ \mathrm{N} = 0.20\ \mu\mathrm{N}$$

directed away from q_1.

(*c*)
$$F_E = Eq_3 = (500\ \mathrm{N/C})(-4.0 \times 10^{-10}\ \mathrm{C}) = -0.20\ \mu\mathrm{N}$$

This force is directed toward q_1.

Q6- In Fig. a proton ($q = +e$, $m = 1.67 \times 10^{-27}$ kg) is shot with speed 2.00×10^5 m/s toward P from A. What will be its speed just before hitting the plate at P?



$$a = \frac{F_E}{m} = \frac{qE}{m} = \frac{(1.60 \times 10^{-19}\ \mathrm{C})(3000\ \mathrm{N/C})}{1.67 \times 10^{-27}\ \mathrm{kg}} = 2.88 \times 10^{11}\ \mathrm{m/s^2}$$

For the problem involving horizontal motion,

$$v_i = 2.00 \times 10^5\ \mathrm{m/s} \qquad x = 0.15\ m \qquad a = 2.88 \times 10^{11}\ \mathrm{m/s^2}$$

We use $v_f^2 = v_i^2 + 2ax$ to find

$$v_f = \sqrt{v_i^2 + 2ax} = \sqrt{(2.00 \times 10^5\ \mathrm{m/s})^2 + (2)(2.88 \times 10^{11}\ \mathrm{m/s^2})(0.15\ \mathrm{m})} = 356\ \mathrm{km/s}$$

Q7- An electron starts from rest and falls through a potential rise of 80 V. What is its final speed?

Positive charges tend to fall through potential drops; negative charges, such as electrons, tend to fall through potential rises.

$$\text{Change in PE}_E = Vq = (80\ \mathrm{V})(-1.6 \times 10^{-19}\ \mathrm{C}) = -1.28 \times 10^{-17}\ \mathrm{J}$$

This lost PE_E appears as KE of the electron:

$$\mathrm{PE}_E \text{ lost} = \text{KE gained}$$

$$1.28 \times 10^{-17}\ \mathrm{J} = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2 = \tfrac{1}{2}mv_f^2 - 0$$

$$v_f = \sqrt{\frac{(1.28 \times 10^{-17}\ \mathrm{J})(2)}{9.1 \times 10^{-31}\ \mathrm{kg}}} = 5.3 \times 10^6\ \mathrm{m/s}$$

Q8- (*a*) What is the absolute potential at each of the following distances from a charge of $2.0\ \mu$C: $r = 10$ cm and $r = 50$ cm? (*b*) How much work is required to carry a $0.05\ \mu$C charge from the point at $r = 50$ cm to that at $r = 10$ cm?

(*a*)
$$V_{10} = k\frac{q}{r} = (9.0 \times 10^9\ \mathrm{N \cdot m^2/C^2})\frac{2.0 \times 10^{-6}\ \mathrm{C}}{0.10\ \mathrm{m}} = 1.8 \times 10^5\ \mathrm{V}$$

$$V_{50} = \frac{10}{50}V_{10} = 36\ \mathrm{kV}$$

(*b*)
$$\text{Work} = q(V_{10} - V_{50}) = (5 \times 10^{-8}\ \mathrm{C})(1.44 \times 10^5\ \mathrm{V}) = 7.2\ \mathrm{mJ}$$

Q10- The following point charges are placed on the x-axis: $+2.0\,\mu$C at $x = 20$ cm, $-3.0\,\mu$C at $x = 30$ cm, $-4.0\,\mu$C at $x = 40$ cm. Find the absolute potential on the axis at $x = 0$.

Potential is a scalar, and so

$$V = k\sum \frac{q_i}{r_i} = (9.0 \times 10^9\,\text{N}\cdot\text{m}^2/\text{C}^2)\left(\frac{2.0 \times 10^{-6}\,\text{C}}{0.20\text{ m}} + \frac{-3.0 \times 10^{-6}\,\text{C}}{0.30\text{ m}} + \frac{-4.0 \times 10^{-6}\,\text{C}}{0.40\text{ m}}\right)$$

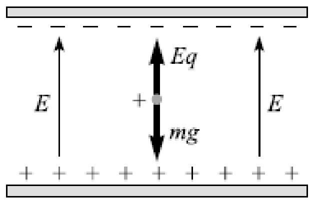
$$= (9.0 \times 10^9\,\text{N}\cdot\text{m}^2/\text{C}^2)(10 \times 10^{-6}\,\text{C/m} - 10 \times 10^{-6}\,\text{C/m} - 10 \times 10^{-6}\,\text{C/m}) = -90\text{ kV}$$

Q11- Four point charges are placed at the four corners of a square that is 30 cm on each side. Find the potential at the center of the square if (a) the four charges are each $+2.0\,\mu$C and (b) two of the four charges are $+2.0\,\mu$C and two are $-2.0\,\mu$C.

(a) $$V = k\sum \frac{q_i}{r_i} = k\frac{\sum q_i}{r} = (9.0 \times 10^9\,\text{N}\cdot\text{m}^2/\text{C}^2)\frac{(4)(2.0 \times 10^{-6}\text{ C})}{(0.30\text{ m})(\cos 45^\circ)} = 3.4 \times 10^5\text{ V}$$

(b) $$V = (9.0 \times 10^9\,\text{N}\cdot\text{m}^2/\text{C}^2)\frac{(2.0 + 2.0 - 2.0 - 2.0) \times 10^{-6}\text{ C}}{(0.30\text{ m})(\cos 45^\circ)} = 0$$

Q12- As shown in Fig. a charged particle remains stationary between the two horizontal charged plates. The plate separation is 2.0 cm, and $m = 4.0 \times 10^{-13}$ kg and $q = 2.4 \times 10^{-18}$ C for the particle. Find the potential difference between the plates.



Since the particle is in equilibrium, the weight of the particle is equal to the upward electrical force. That is,

$$mg = qE$$

or $$E = \frac{mg}{q} = \frac{(4.0 \times 10^{-13}\text{ kg})(9.81\text{ m/s}^2)}{2.4 \times 10^{-18}\text{ C}} = 1.63 \times 10^6\text{ V/m}$$

But for a parallel-plate system,

$$V = Ed = (1.63 \times 10^6\text{ V/m})(0.020\text{ m}) = 33\text{ kV}$$

Q13- Charge is distributed with uniform volume charge density ρ throughout the volume of a sphere of radius R. Determine E everywhere.

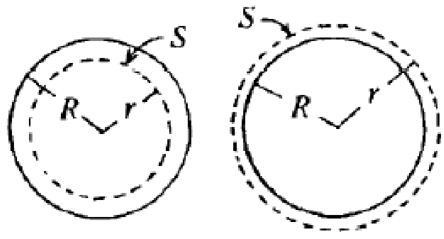
Solution Inside:

$$q_{\text{in}} = \rho\left(\tfrac{4}{3}\pi r^3\right)$$

$$\oint_s \mathbf{E}\cdot d\mathbf{A} = \oint_s E\,dA = E\oint dA = 4\pi r^2 E$$

$$= \frac{q_{in}}{\epsilon_0}, \quad \text{so } 4\pi r^2 E = \frac{\rho\left(\frac{4}{3}\pi r^3\right)}{\epsilon_0}$$

$$E = \frac{\rho r}{3\epsilon_0} \text{ and } r < R$$



Inside Outside

Outside, $q_{\text{in}} = \rho\left(\frac{4}{3}\pi R^3\right)$, so $\oint_s \mathbf{E}\cdot d\mathbf{A} = 4\pi r^2 E = \rho\frac{(4/3\pi R^3)}{\epsilon_0}$

and $E = \frac{\rho}{3\epsilon_0}\frac{R^3}{r^2} = \frac{Q}{4\pi\epsilon_0 r^2}$, $r > R$, where $Q = \rho\left(\frac{4}{3}\pi R^3\right)$

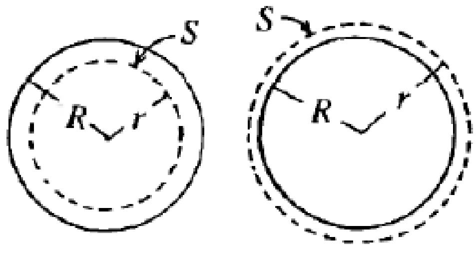
Observe that the field outside a spherical charge distribution is that resulting from a point charge equal to the total charge of the sphere and positioned at the center of the sphere.

Q14- Charge Q is distributed uniformly over a hollow spherical surface of radius R. Determine E inside and outside the sphere.

Solution $\oint \mathbf{E}\cdot d\mathbf{S} = 4\pi r^2 E = \frac{q_{\text{in}}}{\epsilon_0}$

Inside: $q_{\text{in}} = 0$, so $E = 0$, and $r < R$

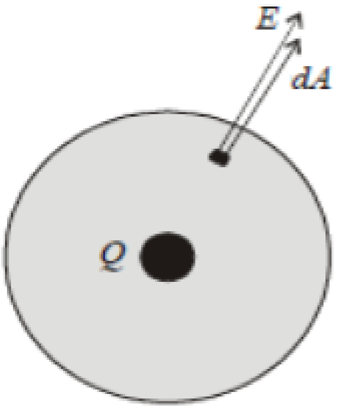
Outside: $q_{\text{in}} = q$, so $E = \frac{q}{4\pi\epsilon_0 r}$, and $r > R$



Inside Outside

Q15- Deduce Coulomb's from Gauss's law

We can deduce Coulomb's law from Gauss's law by assuming a point charge q, to find the electric field at point or points a distance r from the charge we imagine a spherical gaussian surface of radius r and the charge q at its center as shown in figure



$$\oint \vec{E}.d\vec{A} = \frac{q_{in}}{\varepsilon_o}$$

$$\oint E\cos 0 dA = \frac{q_{in}}{\varepsilon_o}$$ Because E is constant for all points on the sphere, it can be factored from the inside of the integral sign, then

$$E\oint dA = \frac{q_{in}}{\varepsilon_o} \quad \Rightarrow \quad EA = \frac{q_{in}}{\varepsilon_o} \quad \Rightarrow \quad E(4\pi r^2) = \frac{q_{in}}{\varepsilon_o}$$

$$\therefore E = \frac{1}{4\pi\varepsilon_o}\frac{q}{r^2}$$

Now put a second point charge q_o at the point, which E is calculated. The magnitude of the electric force that acts on it $F = Eq_o$

$$\therefore F = \frac{1}{4\pi\varepsilon_o}\frac{qq_o}{r^2}$$

Q16- Prove the the energy density in electrostatic field equal :

$$\boxed{W_E = \frac{1}{2}\sum_{k=1}^{n} Q_k V_k} \qquad \text{(in joules)}$$

Suppose we wish to position three point charges Q_1, Q_2, and Q_3 in an initially empty space shown shaded in Figure No work is required to transfer Q_1 from infinity to P_1 because the space is initially charge free and there is no electric field [$W = 0$]. The work done in transferring Q_2 from infinity to P_2 is equal to the product of Q_2 and the potential V_{21} at P_2 due to Q_1. Similarly, the work done in positioning Q_3 at P_3 is equal to $Q_3(V_{32} + V_{31})$, where V_{32} and V_{31} are the potentials at P_3 due to Q_2 and Q_1, respectively. Hence the total work done in positioning the three charges is

$$\begin{aligned} W_E &= W_1 + W_2 + W_3 \\ &= 0 + Q_2V_{21} + Q_3(V_{31} + V_{32}) \end{aligned} \quad \textbf{(1)}$$

If the charges were positioned in reverse order,

$$\begin{aligned} W_E &= W_3 + W_2 + W_1 \\ &= 0 + Q_2V_{23} + Q_1(V_{12} + V_{13}) \end{aligned} \quad \textbf{(2)}$$

where V_{23} is the potential at P_2 due to Q_3, V_{12} and V_{13} are, respectively, the potentials at P_1 due to Q_2 and Q_3. Adding eqs. **(1)** and **(2)** gives

$$\begin{aligned} 2W_E &= Q_1(V_{12} + V_{13}) + Q_2(V_{21} + V_{23}) + Q_3(V_{31} + V_{32}) \\ &= Q_1V_1 + Q_2V_2 + Q_3V_3 \end{aligned}$$

or

$$W_E = \frac{1}{2}(Q_1V_1 + Q_2V_2 + Q_3V_3) \quad \textbf{(3)}$$

where V_1, V_2, and V_3 are total potentials at P_1, P_2, and P_3, respectively. In general, if there are n point charges, eq. **(3)** becomes

$$\boxed{W_E = \frac{1}{2}\sum_{k=1}^{n} Q_kV_k} \qquad \text{(in joules)}$$